Week-03-L-06

Hypothesis Testing

Exercises on Hypothesis testing

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Exercise on Hypothesis Testing

To better understand the problems we do a problem using null hypothesis, which is as follows.

Problem Statement:

- A drip water irrigation machine dispenses water & is working properly when 8 ounces are dripped.
- The average amount dripped in a particular sample of 35 seedlings is 7.91 ounces with a variance of 0.03 ounces squared, s^2.
- Should system be stopped & wait for soaking?

  (The lost streak from a stop is potentially so great that farmer feels that the level of confidence in the analysis should be 99%.)

Solution
Step-1

Set the Null and Alternative Hypothesis.

t or z

- The random variable is the quantity of water dripped in the soil. This is a continuous random variable and the parameter we are interested in is 'the mean'.
- Our hypothesis therefore is about the mean. In this case we are concerned that the machine is not filling properly.
- From what we are told it does not matter if the machine is over-dripping or under-dripping, both seem to be an equally bad error. (Non-directional)

$H_0: \mu = 8$ (Rejected)

$H_a: \mu \neq 8$

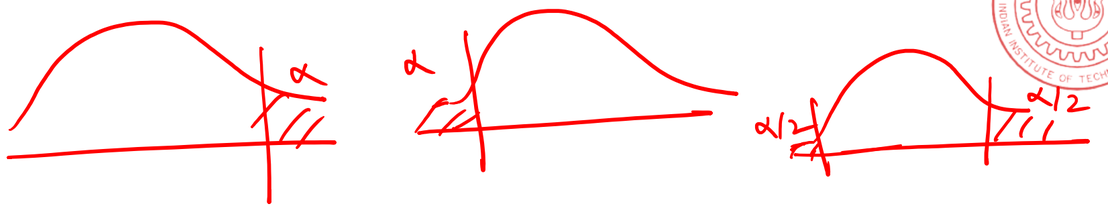
Solution Step-2

Decide the level of significance and draw the graph showing the critical value.

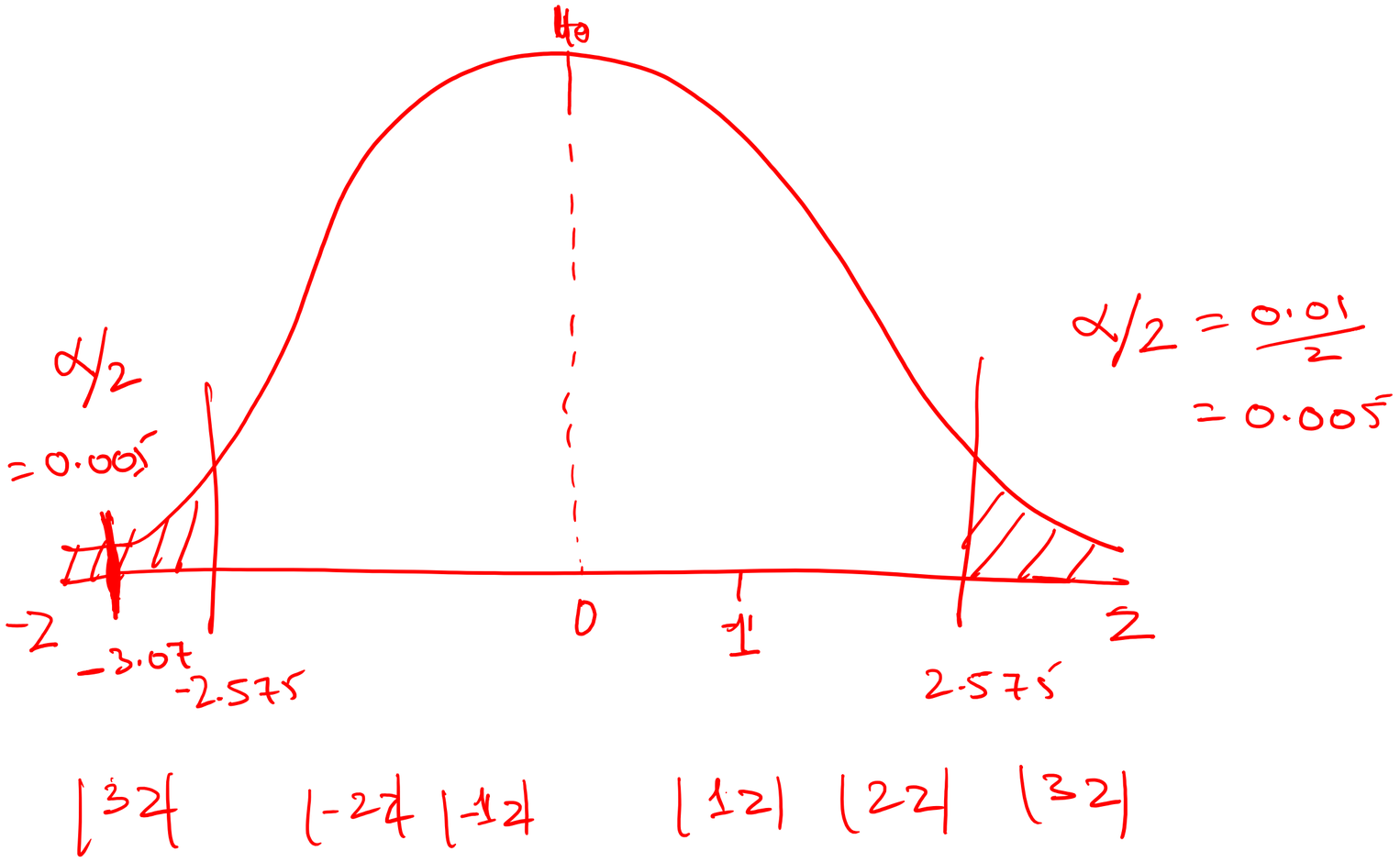
- This problem has already set the level of confidence at 99%. The decision seems an appropriate one and shows the thought process when setting the significance level. (0.01)
- The farmer wants to be very certain, as certain as probability will allow, that he is not shutting down the irrigation machine that is not in need of repair.
- To draw the distribution and the critical value, we need to know which distribution to use.

Solution Step-2

Associated graph



Machine is under-dipping



Solution
Step-3

Calculate sample parameters and the test statistic

- The sample parameters are provided:
  - the sample mean is 7.91
  - the sample variance is .03 $= s^2$
  - the sample size is 35. $= n$

$$S = \sqrt{0.03} = 0.173$$

$$z_C = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

$$Z = \frac{7.91 - 8}{0.173/\sqrt{35}}$$

$$= -3.07$$

Solution Step-4

Compare test statistic and the critical values

- Now we compare the test statistic and the critical value by placing the test statistic on the graph.

- We see that the test statistic is in the tail, decidedly greater than the critical value of 2.575.

Solution Step-5

Reach Conclusion

- Three standard deviations of a test statistic will guarantee that the test will fail. The probability that anything is within three standard deviations is almost zero.
- Actually it is 0.0026 on the normal distribution, which is certainly almost zero in a practical sense.

0.005

Thank you